Statistical Analysis of Antenna Robustness

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Abstract— This paper presents a methodology for the evaluation of antenna susceptibility to various types of random physical damage. A Monte Carlo statistical method is proposed to estimate vital antenna parameters for different damage levels. Virtual experiment is used in order to avoid lengthy experimental trails. Antenna parameters are modeled for each randomly generated damage pattern.

Statistical post processing was used to evaluate the parameters acquired from the antenna modeling software. Computer intensive statistical methods, like the Bootstrap used in present approach, take fewer assumptions than classical statistical methods, and are robust against small deflections from initial assumptions. The BC_a (Bias-Corrected and accelerated) method is used for estimations of confidence intervals.

To illustrate the methodology a comparative study of a damage- inflicted planar dipole and a bowtie antenna was made. The Input Return Loss was the main parameter of interest. The median value was estimated to describe the "typical" behavior of the antenna. With taken assumptions Input Return Loss for the for the Bow- tie antenna is more sensitive to the partial damage than

***Index Terms*—Antennas, Bootstrap, Robustness, Statistics**

I. Introduction

In some cases antennas can be required to work even if its structure has been damaged. One obvious example is military applications, like armor- integrated antennas, where functionality must degrade gracefully even if the antenna has been partially damaged by splinters or fine caliber fire. Another example is low cost transceivers used for Radio Frequency IDentification (RFID) where the need to keep antenna production costs down is accomplished through direct printing of antennas directly on very low-cost substrates, like paper tags [1]. One way to reduce the cost is to use a minimal amount of conducting ink when printing the antenna structure. However, if the antenna substrate is coarse and the paint is applied too thinly, areas where the paint does not fully cover the substrate or is rubbed away might occur. RFID tag antennas must also be reasonably insensitive to careless handling, including tearing and perforations. For both the military applications and the RFID tag antennas the structure has fissures that might affect the antenna performance.

In order to compare antennas from this point of view a Monte Carlo [2] statistical methodology is proposed to estimate vital antenna parameters for different damage levels. Previously the research aimed at investigating structural damage of antennae using classical statistical approach and was primarily directed towards arrays and parabolic antennas [3]-[17]. However, to our knowledge no method has shown the ability to both induce, and statistically describe the effects of any type of random damage to any types of antennas

The use of computer intensive statistical methods holds in perspective to widen the scoop of applications where a statistical methodology is used for antenna analysis whereas these methods do not require the usual idealized models and assumptions. They can also be used for statistical problems that can not be solved by analytical treatment.

II. Methodology

A. General

The approach is a comparison of performance of two or more antennas with inflicted damages, positioned at a random,

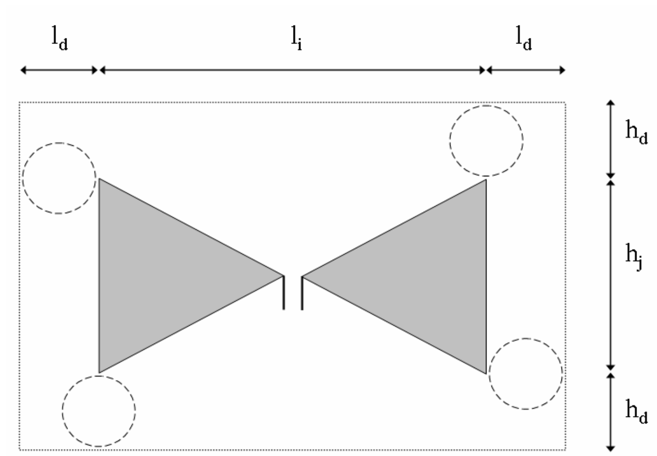

Fig. 1. Bow-tie antenna and circular elementary damages within the imaginary frame.

all over the structure. To swiftly "inflict damage" on a structure and obtain its (degraded) performance commercial electromagnetic simulation software package is used. This software should have a flexible Computer Aided Design (CAD) capability to "build" the antenna structure. The software should also contain some kind of scripting capability

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to allow random positioning of the "CAD holes" in the metal part of the structure ("damages"), simulating the inflicted damage. The antenna simulation data received from the modeling software could be any parameters that particular software package is capable of presenting. Data yielded by the antenna modeling software can be treated with statistical methods to conclude how vulnerable a particular antenna is by itself, or compared to other antennas in a survey.

B. Monte Carlo antenna simulation

A priori knowledge of Maximal Damage Length l_d, Maximal Damage Height l_h, antenna structure Maximal Length l_n and Maximal Height h_n, for each antenna (n = 1...N, where N is the total number of antennas) is required for the assessment Fig.1.

An imaginary circumventing frame with the Frame Length $l_f = 2l_d + l_i$ and Frame Height $h_f = 2l_h + h_j$, were the suffixes i and j indicate the antenna structures with the maximal length and height respectively, is created to limit the spread of the damages. All damage is to be contained within this area. Using the same frame for all antennas in a survey makes it possible to compare the robustness between them. Furthermore, all "CAD holes" that cuts away parts of the antenna metal are considered and the damages that will not hit any part of the antennas are kept at a minimum.

A rather dense meshing is needed for the antenna simulations as two holes can occur close to each other, leaving a very thin bridge of conducting material between them. For each damage case ("hail"), or group of damages, ("buck shot"), the antenna parameters are calculated by the antenna modeling software. The inflicted damage can be represented by several different holes, indentations or anything of damaging nature. To ensure a valid statistical comparison of antennas from this respect, several randomly damaged antennas, for each n, have to be considered.

C. Statistical Post Processing

Considering the wide range of parameters that can be investigated with this method, one does not want to make any more assumptions regarding the statistical models than is absolutely necessary. Computer intensive methods make fewer assumptions than classical methods, and are robust against small deflections from these assumptions [18]. The statistical method of Bootstrapping [19] is especially appealing in case of treatment of statistical data where the underlying statistical distribution is unknown or non-Gaussian [20]. The Bootstrapping methodology makes it unnecessary to take assumptions regarding the inputs and is based on re-sampling of collected data. An asymmetric confidence interval is also desirable to determine how good (or bad) an estimate is since there may be large deviations in one direction if, for instance a parameter goes to zero or infinity when a damage occurs at the antenna feed.

1) Bootstrap

This chapter outlines an application of the bootstrap methodology described in [21]. A random sample $\boldsymbol{x} = (x_1, x_2, ..., x_n)$ from an unknown probability function F is observed and the sought parameter θ is estimated. An estimate of θ is

$$\hat{\theta} = s(\boldsymbol{x}), \tag{2}$$

where $s(\boldsymbol{x})$ is a *statistic* – a function of the sample $\boldsymbol{x}$. If $\hat{F}$ is the empirical distribution with probability $1/n$ on each of the observed x_i, $i = 1, 2, \ldots, n$, a bootstrap sample is defined as a random sample of size n drawn from $\hat{F}$, say $\boldsymbol{x^*} = (x^*_1, x^*_2, ..., x^*_n)$,

$$\hat{F} \rightarrow (x^*_1, x^*_2, ..., x^*_n). \tag{3}$$

In other words, the bootstrap data points $x^*_1, x^*_2, ..., x^*_n$ are a random sample of size n drawn with replacement from the population of n objects $(x_1, x_2, ..., x_n)$. The bootstrap data set $(x^*_1, x^*_2, ..., x^*_n)$ consists of members of the original data set $(x_1, x_2, ..., x_n)$, some appearing zero times, some appearing once, some appearing twice, etc.

The star notation indicates that the bootstrap data set $\boldsymbol{x^*}$ is not the actual data set $\boldsymbol{x}$, but rather a randomized, or *resampled*, version of $\boldsymbol{x}$. The Bootstrap replication of $\hat{\theta}$ for the bootstrap data set $\boldsymbol{x^*}$ is

$$\hat{\theta}^* = s(\boldsymbol{x^*}). \tag{4}$$

In the example below $s(\boldsymbol{x})$ is the median. A selection of B independent bootstrap samples $x^{*1}, x^{*2}, \ldots, x^{*B}$, each consisting of n data values drawn with replacement from $\boldsymbol{x}$ are evaluated with

$$\hat{\theta}^*(b) = s(\boldsymbol{x^{*b}}), \tag{5}$$

b= 1,2,...,B for each bootstrap sample b. The value B is dependent on the type of inference one whishes to perform. For the BC_a method (used below) it is suggested to use B>1000 [22] to reduce the bootstrap sampling error (a higher number makes for smaller errors).

2) Bootstrap confidence interval

The BC_a (Bias-Corrected and accelerated) method [23] for confidence intervals is second order accurate and transformation respecting. This means that for the BC_a method the errors go to zero rapidly in terms of the sample size and that the BC_a endpoints transform correctly for a change of the parameter θ to a function of θ.

An interval of intended coverage $1-2\alpha$ (nominal confidence level) is given by $(\hat{\theta}^{*(\alpha_1)}, \hat{\theta}^{*(\alpha_2)})$, where α is the tail-fraction [24],

$$\alpha_1 = \Phi\left(z_0 + \frac{\hat{z}_0 + z^{(\alpha)}}{1 - \hat{a}\left(\hat{z}_0 + z^{(\alpha)}\right)} \right) \tag{6}$$

and

$$\alpha_2 = \Phi\left(z_0 + \frac{\hat{z}_0 + z^{(1-\alpha)}}{1 - \hat{a}\left(\hat{z}_0 + z^{(1-\alpha)}\right)} \right). \quad (7)$$

$\Phi(\cdot)$ is the standard normal cumulative distribution function where $\hat{z}^{(\alpha)}$ is the 100α-th percentile point of a standard normal distribution and $\hat{z}_0$ is the Bias-correction

$$\hat{z}_0 = \Phi^{(-1)}\left(\frac{\#\left\{\hat{\theta}^*(b) < \hat{\theta}\right\}}{B} \right). \quad (8)$$

(The notation $\#\{x_i < \theta\}$ means the number of x_i s less than θ and $\Phi^{-1}(\cdot)$ is the inverse standard normal cumulative distribution function.)

To determine the acceleration term the so-called *jackknife* technique [25] can be implemented. This method uses samples that leave out one observation. However, when using the median for $s(\boldsymbol{x})$ the jackknife technique is inconsistent. A way to achieve consistency here is to remove more than one observation. The number of sample points left out: d , must be within the interval $\sqrt{n} < d < n$, where n is the total number of observations. Using the bootstrap methodology on the jackknife lessens the burden of drawing all possible combinations of samples. A random sequence of sample points is drawn many times instead. So

$$\hat{a} = \frac{\sum_{s=1}^{m}\left(\hat{\theta}_{(\cdot)} - \hat{\theta}_{(s)}\right)^3}{6\left(\sum_{s=1}^{m}\left(\hat{\theta}_{(\cdot)} - \hat{\theta}_{(s)}\right)^2\right)^{3/2}} \quad (9)$$

is the acceleration term, which relates to the rate of change of the standard error of $\hat{\theta}$ in (2) with respect to the true parameter value θ.

Let $\boldsymbol{x}_{(s)}$ be the original sample with the subset s deleted and m the number of random sequences to be drawn.

$$\hat{\theta}_{(s)} = s\left(\boldsymbol{x}_{(s)}\right) \quad (10)$$

$$\hat{\theta}_{(\cdot)} = \sum_{s=1}^{m} \frac{\hat{\theta}_{(s)}}{m} \quad (11)$$

The above is a bootstrap influenced modification of the jackknife technique for estimating the bias and standard error of an estimate.

III. EXAMPLE

A. Example setup

To illustrate the methodology a comparative study of the random damage inflicted upon a Planar Dipole and a Bow- tie antenna designed for the same frequency was made. These antenna types were chosen due to the simplicity of the shapes and the fact that that they have approximately the same metallic surface area. Both antennas are designed to work (resonating) at 2.45 GHz (a typical RFID frequency is selected) and the parameter investigated was the Input Return Loss. The median value was studied to get the “typical” behavior of the antenna even if there are outliers in the simulation data. The outliers arise when damage occurs in a “vital” part of the antenna, i.e. at the antenna feed. The antennas were modeled as Perfect Electric Conductors (PEC) placed in free space, without any supportive substrate. The Damage Radius was chosen to be about the size of the regular surface roughness for some Printed Circuit Board substrates (see below).

Sheet thickness: 0.1 mm
Damage radius: 0.61 mm (circular hole) (λ/100)
Feed gap: 2.0 mm
The “imaginary” frame: 55.62 x 11.92mm

1) *Bow-tie antenna*

The Bow-tie antenna has an arm length of 20.0 mm and a flare angle of 30 degrees, which gives a maximum height of the structure of 10.7 mm.

2) *Planar Dipole antenna*

The dipole antenna has an arm length of 26.2 mm and a height of 4.1 mm.

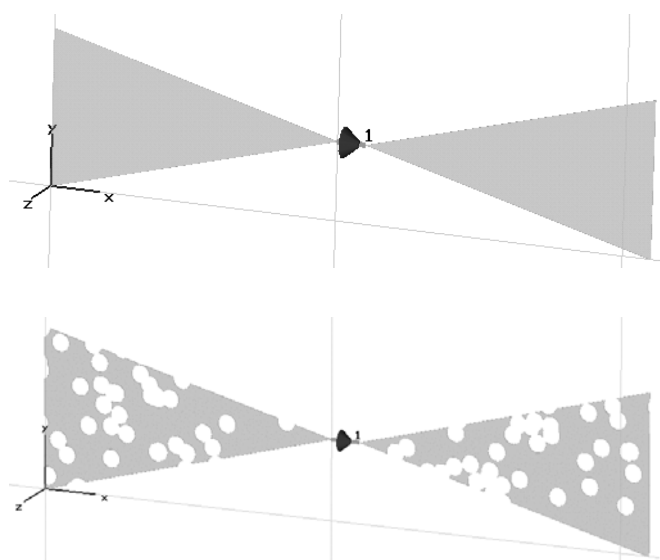

Fig. 2. (a) Undamaged Bow-tie antenna. (b) Bow-tie antenna after 16 “buck shots” with ten “hails” each.

The CST Microwave Studio version 4.0 (MWS), full- 3D time domain EM simulation software [26] was used for antenna modeling. This software allows using Visual Basic scripts [27] to control the program. The damages inflicted on the CAD antenna model were represented by circular holes perforating the structures orthogonally to the planar surface of the antennas. The simulation results were saved as an ASCII-file for subsequent data processing.

The simulation data from MWS was processed in Matlab with the aid of the Matlab Statistics Toolbox. [28] The data transferred from MWS contained number of holes and the antenna Input Return Loss for each "buck shot" of damages. The "buck shot" approach rather than making statistics for one "hail" at a time was chosen due to the larger number of simulations and the dramatically longer estimated computer time required for such task. Matlab was used to calculate the Bootstrap statistics for the two antennas. The Matlab Statistics Toolbox already contains a "Bootstrap" command which helps to reduce the programming effort needed for a Bootstrap analysis.

The change in Input Return Loss was plotted versus the number of "buck shots". This way a comparison of the two antennas with respect to their tolerance to a certain number of damages was illustrated.

B. Example results

Fig 2. shows the Bowtie antenna outline a) before any damage has been inflicted, and b) when the antenna has been damaged. This figure also illustrates why the "imaginary" frame should be larger, than the largest antenna dimension in all directions by at least one damage hole diameter. The "light touches" of some hails, like the one in upper left corner of Fig 2. (b), must be also accounted for in order to ensure a realistic damage case and even spread of the antenna "injuries".

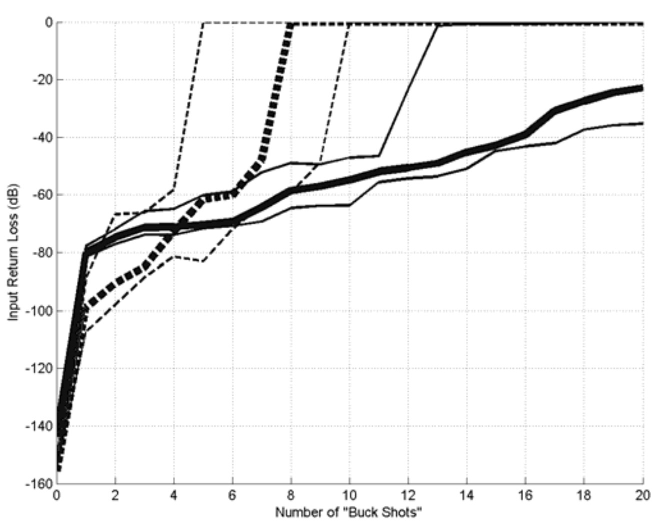

Fig. 3. Input Return Loss (in dB) at 2.45 GHz with a 90% confidence interval for Flat Dipole and Bow- tie antennas respectively after 20 sample structures per antenna. The solid fat line (Dipole) and dotted fat line (Bow-tie) are the estimated values and the solid thin lines (Dipole) and dotted thin lines (Bow-tie) are the upper and lower portion of the confidence interval.

Making the frame any larger will just increase computation time.

The Input Return Loss (in dB, at 2.45 GHz) is plotted for the Dipole and the Bow- tie antenna for an increasing number of "buck shots" is present in Fig. 3.

It can be seen that the confidence area for the Bow- tie (i.e. the area showing the upper and lower limits of the confidence interval) falls more rapidly up zero as compared to the one for the dipole. The conclusion is that Input Return Loss for the Bow- tie is more sensitive to this kind of damage than for the Dipole.

The swift raise in the upper limit of the confidence interval is due to damage that entirely destroys antenna functionality, i.e. damages at the feed. It seems that the Bow-tie antenna feed connected at a sharp apex is more sensitive to damage of this type as compared to the case of the Planar Dipole antenna where feed is connected to a wider strip of the antenna structure. It is also possible to create a narrower confidence interval, by removing outliers in the statistical data, like simulation cases where damage occurs at the feed at one of the first "buck shots". This would however result in some loss of information. A structure with a sensitive feed arrangement would gain more in this exercise, than a robust feed arrangement would and is therefore not used for this analysis where a realistic comparison is needed.

This method simplify the modeling of the damage case to manipulation of the CAD-model whereby the user can simulate the actual physical damage made to the antenna without the need for elaborate statistic model formulation. Also much wider range of antenna parameters can be addressed as compared to previously devised methods. The use of Bootstrap methodology made it possible to formulate a general process for all types of antennas and damage cases.

IV. Conclusion

This paper outlines a promising methodology for evaluating antenna susceptibility to various types of random physical damage. The results are treated by the computer intensive Bootstrap statistical method, thereby reducing or even eliminating the need for modeling and simplifying assumptions previously used for analytical treatment of complex statistics. Damage effect upon various resulting antenna parameters can be studied this way.

More parameters than given in the simple example presented in this paper are needed to give a comprehensive picture of antenna behavior under damage conditions. Generally, the suggested method should be good for realistic antenna comparison when incorporating additional parameters of study.

In the future this methodology will be used with other antenna structures with the aim to find robust antenna designs that is impervious to physical damage.

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